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1100 Seventeenth Street, N.W. Washington, D.C. 20036

SUBJECT: Secular Perturbations of a Near Mars Satellite
Case 710

DATE: July 19, 1968

FROM: S. Bayliss

ABSTRACT

A satellite in a close orbit about Mars is subject to perturbations due to the oblateness of the planet. This paper examines the magnitudes of the secular perturbations of the orbital elements for the proposed orbit of the 1971 Mariner Mars orbiter. The changes are found to be -.166 deg/day for the longitude of the ascending node (Ω) and .0415 deg/day for the argument of periapsis (ω). For the proposed 90 day mission this amounts to values of $\Delta\omega = 3.74°$ and $\Delta\Omega = -14.92°$

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MEMORANDUM FOR FILE

A satellite in a close orbit about Mars is subject to perturbations due to the oblateness of the planet. This paper examines the magnitudes of these perturbations for the proposed orbit of the 1971 Mariner Mars orbiter.

The approximate orbital parameters of the 1971 Mars orbiter are:

Periapsis height = 2000 km

Period = 12 hours

Inclination = 60°

From this, we may calculate

Semi-major axis (a) = 12,690 km

Mean motion (n) = 1.455 (10^{-4}) rad/sec

Eccentricity (e) = .574

Parameter (p) = 8550 km

For Mars, we have [(1)]

Gravitational constant (μ) = 4.293(10^{4}) km^3/sec^2

Oblateness term (J_2) = 1.92 (10^{-3})

Equatorial radius (r_e) = 3410 km

From Kozai [(2)], we find that to first-order in the small parameter J_2, the only secular changes in the orbital elements are given by

$$\frac{d\omega}{dt} = \frac{3r_e^2 J_2 \bar{n}}{4p^2} (5 \cos^2 i - 1)$$

$$\frac{d\Omega}{dt} = \frac{3r_e^2 J_2 \bar{n}}{2p^2} \cos i$$

$$\bar{n} = n + \left[\frac{3nr_e^2 J_2}{2p^2} \sqrt{1-e^2} \left(1-\frac{3}{2}\sin^2 i\right)\right]$$

where i = inclination of orbit to the equatorial plane

ω = argument of periapsis

Ω = longitude of the ascending node

See Figure 1 for an illustration of the orbital geometry. Note that all the elements are mean elements obtained by averaging the instantaneous elements from 0 to 2π in true anomaly (f).

Substituting in the values for the nominal elements of the Mars orbiter and the physical constants for Mars, we get

$$\frac{d\omega}{dt} = .166(5\cos^2 i-1)$$

$$= .0415 \text{ deg/day}$$

$$\frac{d\Omega}{dt} = -.322 \cos i$$

$$= -.166 \text{ deg/day}$$

$$\bar{n} = n[1-6.24(10^{-3})(1-\frac{3}{2}\sin^2 i)]$$

$$= n[1-7.79(10^{-4})]$$

The third equation indicates that the period is slightly longer than the standard elliptic formula would predict. This amounts to an increase in the period of about 34 seconds and would cause significant errors in the time of perigee passage if neglected.

For a 90 day mission, the changes in the ω and Ω amount to

$$\Delta\omega = 3.74°$$

$$\Delta\Omega = -14.92°$$

Note that $\frac{d\omega}{dt} = 0$ at about $i = 63° 26.7'$. For inclinations above this, the line of apsides will regress, while for inclinations below this, it will advance. The equation for $\frac{d\Omega}{dt}$ indicates that the line of nodes always moves in a direction opposite to that of the motion of the satellite.

Kozai(2) also gives expressions for short-periodic terms (arguments of multiples of the true anomaly) and long-periodic terms (arguments of multiples of the argument of periapsis) but, for the case considered, the magnitudes of these perturbations are small (e.g., on the order of a few minutes of arc for ω and Ω) and are therefore neglected.

Stephen Bayliss

S. Bayliss

1014-SB-gml

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2. Kozai, Y., "The Motion of a Close Earth Satellite," _The Astronomical Journal_, November, 1959, Volume 64, No. 9.

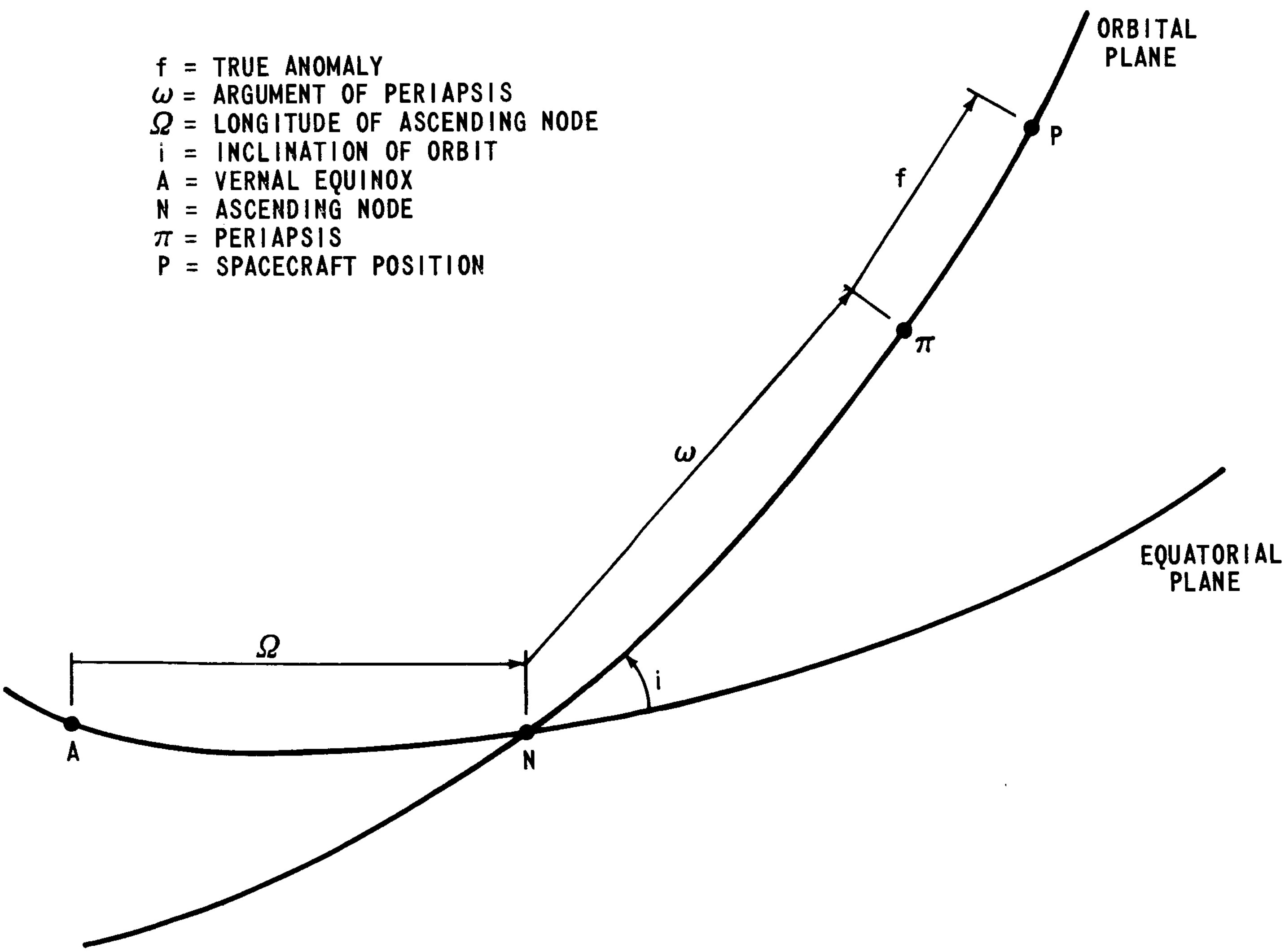

FIGURE I - GEOMETRY OF ANGLE VARIABLES

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